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Reconfiguring a Multi-period Facility Model – An Empirical Test in a Dynamic Setting

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Abstract. Facility location is an important problem faced by companies in many industries. Finding an optimal location for facilities and determining their size involves the consideration of many factors, including proximity to customers and suppliers, availability of skilled employees and support services, and cost-related factors, for example, construction or leasing costs, utility costs, taxes, availability of support services, and others. The demand of the surrounding region plays an important role in location decisions. A high population density may not necessarily cause a proportional demand for products or services. The demography of a region could dictate the demand for products, and this, in turn, affects a facility's size and location. The location of a company's competitors also affects the location of that company's facilities. Another important aspect in facility location modeling is that many models focus on current demand and do not adequately consider future demand. However, while making location decisions in an industry in decline, carefully and accurately considering future demand is especially important, and the question in focus is whether to shrink or close down certain facilities with the objective of keeping a certain market share or maximizing profit, especially in a competitive environment.

This paper develops a model which enables companies to select sites for their businesses according to their strategy. The model analyzes the strategic position of the company and forms a guideline for the decision. It investigates which facilities should be closed, (re)opened, shrunk, or expanded. If facilities are to shrink or expand, the model also determines their new capacities. It depicts the impact on market share and accounts for the costs of closure and reopening. A number of papers deal with location theory and its applications, but few have been written for modeling a competitive environment in the case of declining demand. Existing papers in this area of research are mostly static in nature, do not offer multi-period approaches, nor do they incorporate the behavior of competitors in the market. To demonstrate the validity of the model, it is first solved using a small problem set – three facilities, three demand locations, and three periods – in LINGO solver. To get a better understanding of the model's behavior, several additional scenarios were constructed. First, the number of demand locations was extended to 10. Our findings show that the model presented provides an extension of existing facility location models that can be applied to a variety of location problems in commercial and industry sectors that need to make their decisions considering future periods and competitors. The developed heuristic shows multiple options for solving the problem, including their advantages and disadvantages, respectively. The Java code and LINGO fragments thus developed can be used to provide easy access to related problems.

Keywords: Facility location, single product model, location optimization, dynamic market, competition.

INTRODUCTION

Location problems deal with the allocation of resources in space. There are a number of heuristic solutions organizations can adopt to solve facility location complexities. However, the literature on facility problems is extensive, with a variety of solutions to facility problems. According to Brandeau and Chiu [9], the general location paradigm deals with one or more facilities, called servers, serving a set of customers that are distributed in a specific manner in a region. Location decisions are critical and strategic and must be made thoroughly because, very often, they require high investments [43].

Facility location is an important underlining problem faced by companies across a wide spectrum of industries. Finding an optimal location for facilities and determining their size involves the consideration of many factors, including proximity to customers and suppliers, availability of skilled employees and support services, and cost-related factors, for example, construction or leasing costs, utility costs, taxes, availability of support services, and others.

In addition, the demand for the catchment area plays an important role in location decisions. A high population density may not necessarily cause a proportional demand for products, or services. The demography of a region could dictate the demand for products and this, in turn, affects a facility's size and location. Furthermore, the decision to locate a facility is also influenced by the level of competition within the region since rival players will attempt to locate where there is potential for both profit and market share maximization. Thus, optimal location requires the firm to identify multiple parameters, such as good network connectivity to roads and railway systems, an efficient air transport system, and an overall functional infrastructure.

The location of a company's competitors also affects the location of that company's facilities. Just because a competitor has a facility in a particular location does not necessarily mean the company must also locate its facilities there. Local restrictions, such as laws, subsidies, and taxes play an important role in a facility location decision. Infrastructural aspects also play a role in the location decision. A facility that has good connections to highways, air transportation, and railroads is more attractive to those with infrastructural bottlenecks.

Developed countries may not necessarily have higher demand for products compared to emerging markets. In fact, the opposite could be true. Massive declines in sales have had a deep impact on the economic survival of traditional industries in the last two decades, both in the United States and Europe. In some industries, the focus has now shifted toward the closure of facilities because markets are saturated and new business models have caused changes in business practices. How long to operate a facility in a market that is declining is thus a strategic question. Reports on the closures of drugstores or building centers are part of the daily news in Germany. Furthermore, the growth of e-commerce has changed businesses completely and will dominate the development of business process automation.

Demographic changes, such as declining populations and increased immigration, are causing significant changes in markets and demand, as well as the product mix that companies must offer.

Location decisions are sometimes made based on political considerations rather than careful, rational, and systematic analysis. Another important aspect in facility location modeling is that many models focus on current demand and do not adequately consider future demand. However, while making location decisions in an industry in decline, carefully and accurately considering future demand is especially important, and the question in focus is whether to shrink or close down certain facilities with the objective of keeping a certain market share or maximizing profit, especially in a competitive environment. There is a strategic imperative in such decisions whereby an organization may capture cost savings as a result of closing or shrinking facility locations as well as the critical role in the strategic design of supply chain networks [47]. This is because location decisions are pivotal for a firm's strategic planning because they will have spill-over effects on logistics, customers, and operational activities.

In their work, Correia and Melo [17] introduced an extension of the classical multi-period facility location problem by taking into account different customer segments with distinct sensitivity to delivery lead times. Due to the spatial distribution of customers, decisions are conducted on the basis of discrete facility location models that determine the number, location, and capacity of where facilities should be. The changing market dynamics and business conditions, together with increased cost pressure associated with property acquisition and increased service requirements, compel the firm to restructure its network facilities. This could be associated with the firm aiming to leverage its operational efficiency by taking strategic actions aimed at maintaining a competitive edge through supply chain reconfiguration [61]. Such dynamic supply chain reconfiguration will allow the firm to establish the optimal location and capacity for each of the facilities and use effective links to support transportation networks and effective planning of material flow. This dynamic approach raises key questions about whether to open new geographic facilities, expand the existing ones, contraction of capacities or even complete closure of facilities, in particular those facilities that exhibit poor performance over time.

LITERATURE REVIEW

The topic of facility location has and continues to attract a plethora of research interests and attention in discrete and continuous optimization due to its strategic importance to a firm's survival. Das et al. [18] argue that the primary goal is to optimize at least one economic criterion (e.g., transportation cost, transportation time, revenue, good service, customer coverage, and market share). The facility location literature has used multiple algorithms and models to elucidate what has become a strategic decision for firms.

Others, like Jakubovskis [39], examine a robust optimization (RO) modeling technique that reveals some insights on how firms can strategically conduct capacity planning, confront technology choice problems and facility location challenges. The findings reveal how firms can be able to capture both economies of scale and scope under certain demand realizations.

Location optimization has been studied since the beginning of the 19th century. The early studies focused on a more economic level in the theory of land usage see [58] or in the theory of central locations [13]. Location theory on a business level was formally introduced by Weber [60], and Isard [38] extended Weber's work. Others also started working on location problems by considering the problem of locating two competing vendors along a straight line, e.g., Hotelling [36]. A number of different researchers have studied the location problem from multiple perspectives and have published various papers [1, 2, 5, 12, 15, 20, 21, 23, 24, 27, 30, 31, 36, 49, 51, 57, 60].

LOCATION MODELS

Papers on location theory have been classified by Francis et al. [28]. They distinguish between four classes of problems: continuous planar, discrete planar, mixed planar, and discrete network problems. However, the major planar location problems rely on the assumptions that "demand of customers is represented by a finite set of discrete points and the placement of facilities on the plane" [10]. For this reason, it is extremely difficult and impractical to represent every customer site as a separate demand point. Therefore, decision makers have to aggregate customers by clustering them by postal code, census tracts, etc., but with some aggregation problems [29]. However, most planar location problems rely on the assumption that the demand for customers is represented by a finite set of discrete points [10].

Colombo and Dawid [14] explicitly investigate how firms determine an optimal location choice by accounting for technology spillover within a Cournout oligopoly. Thus, firms can decide to locate as an isolated player or within an industrial cluster, whereby there is a tendency for technology spillover between firms in a spatial setting.

Daskin [19] splits location models into three types of analytical models: continuous models, network models, and discrete models. According to Daskin [19], analytical models are not applicable to certain problems. The main differences according to this classification are to be found in the decision space (market) [32]. Klose and Drexl [43] adopt a clustering approach by distinguishing between continuous location models, network models, and mixed-integer programming models. The latter two types of models are discrete optimization models, so this classification has much in common with the classifications mentioned earlier. They narrow down the mixed-integer programming models into (1) single vs. multi-stage models, (2) uncapacitated vs. capacitated models, (3) single vs. multi-product models, (4) static vs. dynamic models, (5) deterministic vs. stochastic models, (6) models with and without routing options included, and (6) single vs. multi-objective models. In his work, Bieniek [8] presented a note on the facility location problem with stochastic demands.

Traditionally, location problems consider exogenous demand even though the firm may not know the exact distribution of such endogenous demand and how it may affect location choices [6]. Such decisions have a strategic impact on how the choice of location will influence the competitive advantage of the firm. Stochastic and robust optimization approaches are used in the context of facility location problems to deal with demand uncertainty (e.g., [3, 4, 55]).

A further categorization splits location models into static and deterministic, continuous and stochastic problems [48]. Static and deterministic problems can be broken down into median-based-models, covering based-models, center models, and other models. As the covering models focus on designing public services, they might not be practical for business applications, but because those models play an important part in location theory, they will be at least sketched.

The p-center problem locates p facilities with the goal of minimizing the maximum distance between a demand point and its nearest facility. Thus, the p-center problem is also called the minimax problem. It is probably one of the most well-known problems in this category. For business applications, the median problems are much more applicable. Among these is the p-median model. The p-median problem, introduced by Hakimi [33], minimizes the total demand-weighted travel distance between the demand points and the facilities. In contrast to the p-median model, the covering problem addresses a certain service level. A demand is covered if it can be served at a certain time or distance. This measure describes the desired service level. The objective of the covering problem is to minimize the total cost of reaching that service level.

Uncertainty about the future is part of any model as it is the strategic nature of facility location problems [48]. Dynamic location models consider an extended planning horizon and generate robust location decisions. Longer planning horizons and uncertainties in facility location are addressed by dynamic location models. Although the dynamic location models consider an extended planning horizon, they are static in nature because all parameters are defined in advance and possible parameter changes in later periods are not considered. For example, in their study, Silva et al. [56] examine the Dynamic Facility Location Problem with Modular Capacities (DFLPM) by generalizing location problems and solving them using customized linear-based heuristics for each given scenario and the relative cost structures.

According to Correia and Melo [16], dynamic facility location asserts that parameters in facility location problems shift over the planning horizon. This compels the firm to periodically review facility location decisions over time to make an adaptation that provides a fit with the changes in distribution network and demand patterns [40].

Stochastic models assume that the input parameters are mostly unknown in real-world applications. Owen and Daskin [48] split the models into two different approaches, which are the probabilistic approach and the scenario planning approach. The former uses stochastic tools, the latter considers scenarios in a what-if setting. Brandeau and Chiu [9] presented an overview of representative problems and established a classification scheme, which appears to be more general. Their work includes a survey of more than 50 studies with problems in location theory. Their classification is divided into three characteristics, which are the objective of the model, the decision variables, and the system parameters.

Hamacher and Nickel [34] introduced a 5-parameter classification using a formalization based on queuing theory. It takes five parameters into account and was built on the research of Eiselt et al. [26], Eiselt and Laporte [25], who focused on competitive location models. They indicate that their taxonomy has been a useful tool in designing and structuring lectures and research papers. The benefit of their classification is that not only classes of specific location problems are described [34]. The taxonomy uses five parameters to describe the underlying problem. The structure reminds one of Kendall's notation used in queuing theory [41]. These parameters are P1: information about the number and type of new facilities, P2: Location model type in relation to the decision space, P3: Detailed description of the specific location model, P4: The relation between new and existing facilities, and P5: Detailed description of the objective function.

A subset of symbols for each position is introduced with respect to continuous, network, and discrete location models. Although the classification can be applied to a large variety of location models, Haase and Hoppe [32] expanded the scheme by including additional parameters for competitive location models. The resulting taxonomy consists of six parameters using a similar notation as described earlier, which are: characteristics of competition, characteristics of decision space, modeling of demand or market share, pricing strategy, objective, and additional parameters.

Haase and Hoppe [32] mention that only a few authors have discussed the shrinking or closing of facilities.

Some more specialized classification schemes can be found by Handler and Mirchandani [35] in a 4-position scheme, applicable to network location models with objective functions of the center type. Carrizosa et al. [11] developed a 6-position scheme for the classification of planar models, referring mainly to the Weber problem.

Very little attention has been paid in the literature to the closing or relocation of facilities. Revelle et al. [50] give a first overview of papers both for a competitive and a noncompetitive setting, but do not set up a classification scheme. Finally, they end up with two static models for the shrinkage problem.

A Classification Scheme for Facility Location Models Focusing on Opening, Closing, and Capacity Decisions Incorporating Competition

According to the different taxonomies described earlier, it is necessary to set up a scheme that addresses the objective of the model in this paper to get an expanded and more diversified view of the body of literature. The classification scheme includes the following parameters: decision space, model objective, distance measuring, time, competition, consumer and demand, single/multiple products, homogenous/heterogeneous facilities, constraints, opening and closing, and solution methods (linear programming or heuristics).

Shrinking Facility Size and Closing

There have been limited studies conducted on the topic of shrinking and closing of facilities (for example, [7, 53]). Most studies have extensively focused on location problems, and resulting in a gap in literature. A good starting point is to be found in a model proposed by Klincewicz et al. [42]. They propose a discrete network model with the objective of minimizing the total discounted costs over a given planning horizon, which indicates that it is a dynamic approach. Instead of distances, costs play an important part in the model. The model allows the use of concave operating costs. Competitors are not considered, though it is to be assumed that the model should be applied to business and not to public services. The demands are determined by arbitrary demand patterns. This allows one to model different demands at different periods. As it is possible to shrink or expand facilities in this model, this allows the modeling of facilities of different sizes. The constraints in the model focus primarily on the capacity of the facilities and the demand. To incorporate the arbitrary demand patterns, Klincewicz et al. [42] allowed both contraction and expansion as well as closing and opening of locations. To solve the model, the authors start with a myopic initial solution and then apply a heuristic.

Melachrinoudis and Min [46] focused on supply chain management objectives. They consider the phase-out and relocation situation of a hybrid, two-echelon plant/warehousing facility with respect to changes in business environments. The decision space is discrete as the possible locations are known in advance. The objective of the model is to maximize the total profit in a given time horizon with respect to location incentives. Distances are measured in minutes. Competitors are not considered. The demands of the customers in certain periods are known as well and are not influenced by the location decision. Melachrinoudis and Min [46] do not consider special product types. Instead, they use units to model both capacity and demand. Because the goal is to relocate the facility, there is no difference made between homogenous and heterogeneous facilities. The major constraints of the model pertain to production capacity and demand satisfaction. The model either closes or opens exactly one facility. The model then computes where and when to open and close. The model is solved by using LINGO.

Bhaumik [7] examines existing facilities and the corresponding network and finds that they do impose additional constraints on the closure or elimination of facilities and terms this as a "facilities delocation problem." In this study, the delocation problem is formulated as an integer linear programming.

Wang et al. [59] consider a budget-constrained location problem with the opening and closing of facilities. The model is formulated in a discrete decision space. A set of nodes of existing and possible locations is given. The objective of the model is to minimize the total weighted travel distance for customers. Distance is measured in Euclidean space, but the distances are also weighted such that a certain demand node could become more important than another. The model is a single planning period model. Because the objective is to minimize the total weighted travel distance, the customers' competitors are not considered in the model. The consumers are modeled using demand nodes. A consumer can be ranked higher by assigning a larger weight. As capacity does not play a part in the model, demand is not considered in the case of units. All facilities serve the same product and are equal. The model employs two major constraints: the first is a budget constraint for facility opening and closing, and the second is the total number of open facilities desired. Facilities may be closed down or be open. As the problem is NP-hard, the authors developed three heuristics.

Zhang and Rushton [62] developed a model that uses a discrete decision space to optimize the size and location of facilities in a competitive environment with the objective of maximizing the spatial utility of users. Distances are measured in Euclidean space. The authors consider only one period. In contrast to the models mentioned previously, the location of competitors is considered. The demand is also known. Zhang and Rushton [62] consider a single product. While facilities can differ in size, larger branches attract more customers. The model incorporates such constraints as budget and size. Locations can either be open or closed. Heuristics are used to solve the model.

ReVelle et al. [50] considered closing facilities in their model. They propose two models, one for firms with competition and another for firms without competition. As this paper will provide a model for firms with competition, only the characteristics of the former are described. The model uses a decision space in a network of discrete points.

The assumption is to retain a given number of facilities, losing as little market share to the competitors as possible. Thus, the model objective is to minimize the market share lost. Distances are Euclidean and the model does not consider multiple periods but takes competition into account. A demand point is lost to a competitor when the distance between one's own nearest facility is greater than the distance of the nearest competitor. The demand is also known in advance. The model uses both single products and homogenous facilities. The authors considered only the closure of a fixed number of facilities. The model is solved using LINGO.

Bi-level models are competitive by nature and incorporate the reaction of the follower. A first model deals with the problem of locating new facilities in a market where competitors are already operating [44]. The new facilities should be such that the profit is maximized. For this purpose, the authors introduce an attraction index. According to the bi-level approach, the competitor can react by adjusting the attractiveness of existing facilities or by opening or closing others to maximize the profit. The distance measure is Euclidean. Multiple periods are not considered explicitly due to the bi-level approach. The consumer's demand for a homogeneous product is modeled by the buying power at the demand points. Because it is possible to adjust the attractiveness of each facility, the model works with heterogeneous facilities. In this model, it is not possible to adjust the attractiveness. For the existing companies in the market, it is possible to open or close facilities. The authors use a heuristic approach to solve the model.

Küçükaydin et al. [45] proposed another, similar model, allowing the adjustment of the attractiveness of the follower but not allowing the closing or opening of facilities. The assignment of demand points to certain facilities is determined by the attraction index.

A further bi-level approach is proposed by Drezner and Drezner [21]. They considered this a model for a company entering a market knowing that a future competitor is expected to enter the market. They state that the location of one's own facility should be chosen such that it is optimal after the market entry of a new competitor. The model objective is to maximize the market share. The model uses a continuous decision space with distance correction to ensure that a facility will not be located directly on a demand point. The distance measure is Euclidean. In this bi-level approach, only single periods are considered. Demand is measured in buying power and is aggregated at prespecified demand points. The model does not consider multiple products. As the model deals with several existing facilities having different attractiveness levels, the facilities are completely heterogeneous. It is not possible to influence the attractiveness level in the model directly.

The model does not incorporate closing or opening decisions. The authors introduce three solution approaches: the brute force approach; the pseudo-mathematical programming approach; and the gradient search approach.

A paper for bank networks was published by Ruiz-Hermández et al. [54]. The paper contains closing down as well as long-term operations costs. It also assumes that it might sometimes be necessary to resize branches. The model is set up for a competitive environment and takes a ceding market share into account. Distances are Euclidean. The model does not include multiple periods. The customers are known, capacities may vary at the locations, and facilities can have different sizes. The problem is solved using the CPLEX algorithm.

A paper by Bhaumik [7] introduces the de-location problem with the objective of closing a prespecified number of facilities and assigning demands to the remaining facilities. The objective of the model is to minimize the total cost of serving the demand nodes, assuming reassignment might cause a higher distribution cost. In the model, distances are not explicitly considered, but costs are given for all assignment pairs. It is assumed that the cost is based on distance. In his paper, Bhaumik [7] did not extend the model to multiple periods, but the model could be computed period by period. Competition is not considered as the focus is on cost minimization. The demands of the nodes are not known. Facilities in the model can be assumed to be homogenous because Bhaumik [7] did not consider any differences in terms of operational costs or attractiveness. The model postulates that demand is not lost through the closure of facilities. The only impact is that there will be higher distribution costs due to the new assignments. A linear solver was used to run the model.

METHODOLOGY

Models for Multi-period for Facility Location

Most location models in the literature do not consider either multiple periods or neglect competition in the model. The model introduced in this study attempts to overcome this limitation. It is based on a competitive, multi-period environment and not only takes downsizing or closing of certain facilities into consideration but also assumes that a downsizing or closing of one facility might require the expansion of others. Contraction of an existing facility is also being considered as an option. Furthermore, competitors are explicitly incorporated into our model. The demands are assumed to vary over a chosen time horizon.

First, the model with a single product is demonstrated. Second, the model is then extended by allowing multiple products as well as the addition of several constraints, because the single product assumption limits the applicability. Both models are solved and computational results are provided.

Model 1

The first proposed model considers an existing set of facilities operating in a competitive environment in a market with discrete demand points. The model considers two different scenarios. First, the demand is known for all periods in advance. This is solved using a robust approach. Second, a method for solving the model period by period is used, because demand can only be predicted more accurately for the next period. The two approaches are then compared. Operating a facility incurs a fixed cost. Closing and opening of facilities will incur a cost just like expansion or contraction. Finally, the model takes the competitive environment into consideration. This is done by using an attractiveness parameter, which indicates the attractiveness of a certain facility. The model postulates that the facility with the highest attraction will be assigned to a demand point if it is open. The highest attractiveness might also be at a competitor's site. The prices for the homogenous product can differ between the facilities as well as between periods. A facility will be operating at a certain capacity in each period.

MATHEMATICAL FORMULATION

Indexes

i Set of facilities
j Set of demands
t Set of periods

Model Parameters

$initialCapacity_i$	The initial capacity of facility i in period 1
$initialOpen_i$	The initial status of facility i in period 1
$price_{it}$	The price for selling a single unit at facility i in period t
$variableCost_{it}$	The unit cost for selling a single unit at facility i in period t
$fixCost_{it}$	The cost for having open facility i in period t
$openingCost_{it}$	The cost for opening facility i in period t
$closingCost_{it}$	The cost for closing facility i in period t
$attraction_{ijt}$	The attraction value of a company's own facility i with respect to demand node j in period t
$competitorsAttraction_{jt}$	The highest attraction of competitors for demand node j in period t
$expansionCost_{it}$	Fixed cost for expansion of facility i in period t

$shrinkageCost_{it}$	Fixed cost for contraction of facility i in period t
$unitExpansionCost_{it}$	Unit (variable) cost for expansion of facility i in period t
$unitShrinkageCost_{it}$	Unit (variable) cost for contraction of facility i in period t
$demand_{jt}$	The demand at demand node j in period t

$$assignment_{ijt} = \begin{cases} 1, & \text{if } attraction_{ijt} \geq competitorsAttraction_{jt} \\ 0, & else \end{cases}$$

This parameter can be set to 1 if the attractiveness of a company's own facility i relative to a demand node j is greater or equal to the highest attractiveness of the competitor for the same node j in period t. It is used for the assignment of facilities to a demand node.

Model Variables

$$open_{it} = \begin{cases} 1, & \text{if facility i remains open in t} \\ 0, & \text{if facility i is closed in t} \end{cases}$$

$$opened_{it} = \begin{cases} 1, & \text{if faciltiy i is opened in t} \\ 0, & else \end{cases}$$

$$closed_{it} = \begin{cases} 1, & \text{if faciltiy i is closed in t} \\ 0, & else \end{cases}$$

$$expanded_{it} = \begin{cases} 1, & \text{if facility i is expanded in t} \\ 0, & else \end{cases}$$

$$shrunk_{it} = \begin{cases} 1, & \text{if facility i is shrunk in t} \\ 0, & else \end{cases}$$

$uPlus_{it}$	Expansion in units at facility i in t
$uMinus_{it}$	Contraction in units at facility i in t
$totalCapacity_{it}$	Maximum capacity in units at i in period t
$usedCapacity_{it}$	Capacity utilized in units at i in period t

Objective Function

$$\begin{aligned} Max \sum_i \sum_t (&usedCapacity_{it} * (price_{it} - variableCost_{it}) \\ &- fixCost_{it} * open_{it} - closingCost_{it} * closed_{it} \\ &- openingCost_{it} * opened_{it}) \\ &- \sum_i \sum_t expansionCost_{it} * expanded_{it} \\ &- \sum_i \sum_t unitExpansionCost_{it} * uPlus_{it} \\ &- \sum_i \sum_t shrinkageCost_{it} * shrunk_{it} \\ &- \sum_i \sum_t unitShrinkageCost_{it} * uMinus_{it} \end{aligned}$$

Subject to

1. $open_{i1} = initialOpen_i, \forall i$
2. $totalCapacity_{i1} = initialCapacity_i, \forall i$
3. $usedCapacity_{it} \leq totalCapacity_{it}, \forall i, t$
4. $usedCapacity_{it} \leq \sum_j assignment_{ijt} * demand_{jt}, \forall i, t$
5. $totalCapacity_{it+1} = totalCapacity_{it} + uPlus_{it} - uMinus_{it}, \forall i, t$
6. $open_{it+1} = open_{it} + opened_{it} - closed_{it}, \forall i, t$
7. $M * expanded_{it} \geq uPlus_{it}, \forall i, t$
8. $M * shrunk_{it} \geq uMinus_{it}, \forall i, t$
9. $totalCapacity_{it} \geq 0, \forall i, t$
10. $totalCapacity_{it} \geq uMinus_{it}, \forall i, t$
11. $expanded_{it} \leq open_{it}, \forall i, t$
12. $shrunk_{it} \leq open_{it}, \forall i, t$
13. $M * open_{it} \geq usedCapacity_{it}, \forall i, t$
14. $\sum_i assignment_{ijt} \leq 1, \forall j, t$
15. $assignement_{ijt}, expanded_{it}, shrunk_{it}, open_{it}, opened_{it}, closed_{it} \in \{0; 1\}, \forall i, j, t$
16. $uPlus_{it}$, $uMinus_{it}$, $totalCapacity_{it}$, $usedCapacity_{it} \in \mathbb{N}$, $\forall\, i, t$

Explanation

The objective function maximizes the profit. It consists of six terms. The first term computes the net profit. The second term includes the fixed cost component when a facility is open. The third and fourth terms are for assigning a closing or opening cost that occurs when a facility is opened or closed, respectively. The last two terms consider the expansion or contraction costs. Constraints (1) and (2) are used to specify the open or closed status of each facility as well as the initial capacity. The usage must never be greater than the capacity. This is ensured by constraint (3). Constraint (4) addresses the fact that the usage cannot exceed the assigned demands. The capacity of the facilities might vary over time. Constraint (5) computes the capacity for the following period by adding the units of expansion or subtracting the units of shrinkage from the current capacity. Constraint (6) is a key constraint and is used for setting the status of a facility to open or close. Constraint (7) sets $expanded_{it}$ to 1 if the facility is expanded. Constraint (8) does the same for the contraction case. Constraint (9) ensures that the capacity will never become negative, and constraint (10) states that the capacity will always be greater or equal to a potential contraction. Constraints (11) and (12) assert that only open facilities can be expanded or contracted, respectively. Finally, constraint (13) ensures that capacity will only be used if a facility is open and constraint (14) postulates assigning a certain demand only to one of the available facilities and only once.

Computational Results

The model was solved with five different settings and a LINGO solver.

Performed Checks

The model was tested with two additional data sets. Again, the solutions were obtained in less than 1 second using the same software and hardware. Several tests were applied to confirm that the model is working correctly. All tests were conducted on all three different data sets and are listed hereunder.

1. Test: If $assignment_{ijt} = 1$, is $usedCapacity_{it} > 0$?

   This test was conducted to confirm that a facility only has used capacity if the corresponding assignment variable is set to 1.
2. If $assignemnt_{ijt} = 1$, is $attraction_{ijt} \geq competitors\ Attraction_{jt}$?

   This condition states that an assignment will only be made if the attraction of the own facility with respect to demand point is greater or equal to the attraction of all competitors in the period t.
3. If $expanded_{it} = 1$, is $uPlus_{it} > 0$?

   If a facility is expanded, the variable $uPlus$ indicating the amount of expansion must be positive.
4. If $shrunk_{it} = 1$, is $uMinus > 0$?

   If a facility is contracted, the variable $uMinus$ indicating the amount of contraction needs to be positive.
5. If $expanded_{it} = 1$, is $capacity_{it+1} = capacity_{it} + uPlus_{it}$?

   If a facility is expanded in period t, the capacity of the facility in the preceding period must be expanded by $uPlus_{it}$ units.
6. If $shrunk_{it} = 1$, is $capacity_{it+1} = capacity_{it} - uMinus_{it}$?

   If a facility is contracted in period t, the capacity of the facility in the preceding period must be reduced by $uMinus_{it}$ units.
7. If $closed_{it} = 1$, is $open_{it+1} = 0$?

   If a facility is closed at the end of period t, it must be closed in the preceding period.
8. If $opened_{it} = 1$, is $open_{it+1} = 1$?

   If a facility is opened at the end of period t, it must be open in the preceding period.
9. If $used_{it} > 0$, is $open_{it} = 1$?

   This test was conducted to confirm that a facility only has used capacity if it is open.

All the tests showed positive results and confirmed that the model was working in the expected way. The only remark that must be made here is that "M" is an arbitrarily large positive number, larger than the highest potential capacity. The highest potential capacity will never exceed the demand at all demand points in any period.

Extension of Model 1

To get a better understanding of the model's behavior, several additional scenarios were constructed. First, the number of demand locations was extended to 10. The problem was solved quickly. Second, the periods were extended to 10. Again, a solution to the problem could be obtained in about 1 second. Third, the number of facilities was also extended to 10. For the first set of data, the model could not be solved even after x hours of computation time. For two subsequent data sets, the optimal solution could be determined in 2 seconds (Table 1).

Another larger data set with 100 facilities, 100 demand points, and 100 periods was also generated. The complexity of the data increased exponentially, and the model with 1,089,800 variables was solved in about 2.5 minutes.

Table 1 summarizes the solution times for the multiple datasets that were used to solve Model 1.

To gauge the problem size that can be solved using standard software, several problem instances were generated. The 100 × 100 × 100 scenario could not be solved within a reasonable time. A 150 × 150 × 150 scenario with more than 3.5 million variables resulted in a LINGO buffer error (Table 2).

Retrieving the Relaxed Solution From the Model

To obtain an upper bound for the solution of the model, all binary and integer constraints were removed from the model. The following results were obtained:

The results of the objective function values (OFVs) show that the relaxed model solutions provide an upper bound that is relatively close to the OFV of the optimal solution for the first two (smaller) problem instances.

Table 1. Variables and runtimes for different datasets.

Dataset	Number of Variables	Elapsed Runtime
3 × 3 × 3	102	<1 sec.
5 × 5 × 5	340	<1 sec
10 × 10 × 10*	1,880	30–250 sec.
100 × 100 × 100	1,089,800	n.a.

*Depending on the data.

Source: Authors.

Table 2. Objective function values for different model sizes both for mixed integer model and relaxed model.

	Objective Function Value	
Data Set	Regular Model	Relaxed Model
5 × 5 × 5	5.50502E08	5.5116E8
10 × 10 × 10	4.18601E08	4.18671E8
100 × 100 × 100[1]	2.05094E11	1.10398E12

[1]Objective function value of a feasible but not of the optimal solution.

Source: Authors.

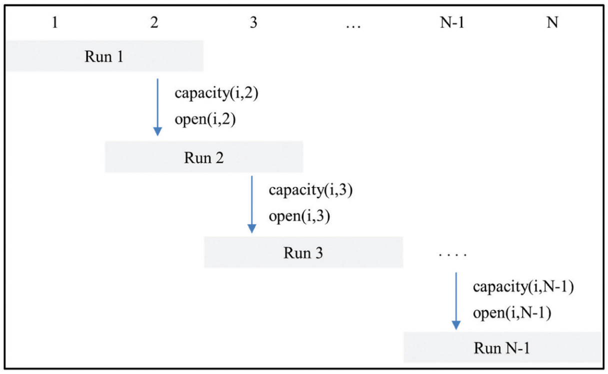

Figure 1. Dynamic solution approach.*Source*: Authors.

Solving the Model in Dynamic Manner

It is relatively easier to forecast demand in the short term than in the long term. Thus, the model was also solved using a dynamic approach; this means the problem is solved for each period considering only the demand for that period. A dynamic solution will result in more frequent opening and closing of facilities due to the myopic approach. An analysis was performed to determine whether a robust solution (solving the model once for the entire planning horizon, even though the demand forecasts, especially for the later periods in the planning horizon, may be inaccurate) or a dynamic approach (solving the model once in each period and using that solution and the next period's forecast demand to determine the next period's solution) will result in a better solution. With respect to the model and its mathematical formulation, a single period-based approach was not implementable as the open status and current capacity are always known for the initial period. Thus, the consecutive period was included in the model and all variables were determined for that period. After obtaining the variable values, the next run was begun using the just generated variable values as initial parameters. Figure 1 demonstrates the procedure.

The dynamic solution was implemented using LINGO scripting. The scripting made it possible to change the relevant variables in the model.

Several checks were performed to explore the models' behavior and to detect the differences between the two approaches (robust versus dynamic).

1. Demand data constant over time horizon

The two approaches show a difference both in terms of capacity and the opening status of facility 1. The robust solution contains an extension of the capacities; the dynamic one keeps the capacity of the initial level for all periods due to the fact that the expansion cost cannot be offset by net profit from one period. The opening status is the same for both models.

2. Variable demand data

Demand can be estimated more accurately in the short-run, but it is difficult to do so for periods far into the future due to a variety of factors, including changes in trends, customer preferences, seasonality, and others. Because the dynamic model only considers the current and next two periods, the solution provided by the two models will be different. The dynamic approach closes a facility immediately if there is no demand in the next period and if the fixed costs are significant. A disadvantage of the dynamic model is that it makes myopic decisions based on the next period's demand. For example, it may close a facility because there is no demand in the next period and open it again in a subsequent period when demand is high. Once it is closed, it will only be opened if the costs can be offset by demand in the next period. The robust approach shows a different solution because demand for multiple periods into the future is assumed to be known. A comparison of objective function values is difficult as the dynamic approach always generates an objective value for two consecutive periods. Variability in net profit. A variability in net profit shows an expansion of capacity in the robust approach. The facility is closed down when net profit decreases and reopened when net profit increases again. The dynamic approach keeps the facilities closed. Lowering opening and closing costs influences the behavior. If the opening and closing costs are relatively low, facilities will be closed and reopened. Demand is constantly decreasing. The behavior of the model in terms of constantly decreasing demand again depends on the net profIt. If the net profit is relatively high, the robust model extends the capacity to be able to satisfy higher demand in the earlier periods. All facilities are kept open throughout the entire planning horizon. If net profit is relatively low, then those facilities with the highest costs are closed first. The dynamic model shows different behavior. Though there is a higher demand than capacity at the beginning, no expansion is made, even in terms of higher net profit. If net profit decreases, the model closes only that facility with the smallest net profit first. Later, all facilities will be closed at the same time.

In a nutshell, the dynamic model is much more sensitive toward variability in demand. The better knowledge of future demands will enable us to satisfy higher demands in earlier periods when using the robust approach.

CONCLUSION

The models presented in this study provide an extension of existing facility location models that can be applied to a variety of location problems in commercial and industry sectors that need to make decisions considering future periods and competitors. An initial model focusing on a single product was developed. The model was extended by including multiple products. The developed heuristic shows multiple options for solving the problem, including their advantages and disadvantages, respectively. The Java code and LINGO fragments thus developed can be used to provide easy access to related problems.

The research in this paper gives direction to further topics that are of great interest and can be very useful for the daily application of the model. The attraction parameter plays a central role in the assignment of a facility. The model takes it as a given parameter. The model can be extended by setting up an attraction function which incorporates additional parameters and may change over time, depending on the decisions made by the model. A utility function can be used for modeling purposes. The current model has several constraints. With respect to certain applications, additional constraints may become necessary, such as subsidies, taxes, space limitations, laws, etc. The development of these constraints will make the model more realistic, but for every problem, the model needs to be adjusted to the relevant use case.

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