For Investment Geometric Problems

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Abstract. Transformation of a plane called inversion is a technique used in geometric problems of special importance. An inversion is a mapping of a plane such that a set of directions and a circle maps to the same set, and in doing so it can map a line either to a line or to a circle, and it can also map a circle to either a line or a circle.

Keywords: Inversion, Euler’s circle, Feuerbach’s theorem, Apollonius problem.

INTRODUCTION

Often, when solving geometric tasks, we use one of the transformations of the plane. In this article, we will get acquainted with inversion, symmetry with respect to the circle, a mapping which in many respects is analogous to symmetry with respect to the direction. Examples of inversion are not often found in nature, but as a relatively young method, it makes many geometric problems easier and is very useful even in problems of mechanical nature.

In this article, the conclusions on geometric inversion are presented, with which all the mentioned mathematicians agreed, and at the end, discussions about its application through interesting tasks and its use in the real world are provided.

PROPERTIES OF INVERSION

In this section, we will study the significant properties of inversions. We will also consider cases such as when the inversion maps direction to direction, direction to circle, circle to direction, and circle to circle. We have listed some specific inversion maps as follows.

Theorem 1:

Let $p \subset M$ be the direction through the center $O$ of inversion $I_O$, then $I_O (p\setminus\{O\}) = p\setminus\{O\}$.

Proof:

The proof directly follows from the fact that $O, T,$ and $T'$ must be collinear points and that the $I_O$ is a bijection.

Theorem 2:

Let the pairs $A, A'$ and $B, B'$ belong to the inverse points of $I_O$ with the circle of the invertebrate $k(O, R)$. Then

$$\angle OAB = \angle OB'A' \quad \text{e.,} \quad \angle OBA = \angle OA'B'$$

Proof:

Let $A$ and $A'$ be associated points of inverse $I_O$, then $|OA| \cdot |OA'| = R^2$ (Figure 1). Analogously, $|OB| \cdot |OB'| = R^2$; however,

Since $\angle AOB = \angle B'O'A'$, $\Delta OAB \sim \Delta O'B'A'$, from which we get $\angle OAB = \angle O'B'A'$ and $\angle OBA = \angle O'A'B'$.

The following assertion is the immediate corollary of the theorem.

**Corollary:**

The inversion is determined by the inversion center and the pair of associates.

**Theorem 3:**

The direction $p$ which does not pass through the center $O$ of inversion $I_O$ is mapped into a circle which passes through the center $O$ of inversion.

**Proof:**

Let $p \subset (M \setminus \{O\})$ be some direction (Figure 2). Make sure that you’re okay with the spu wall from $O$ to $p$, and $N' = I_O(N)$. Let $T \subset p$ be any of the directions $p$, and $T' = I_O(T)$.

**Figure 2.** Direction that does not pass through the center of inversion is a circle.

Theorem 2, $\angle ONT = \angle OT'N' = 90^\circ$. From this, according to the turn of Tales’ teaching, it follows that $T'$ is located on the circle $p'$ with $ON'$ as the diameter. So, each of these directions, $p$, is mapped in that circle, $p'$. As soon as $I_O$ is a bounce, the claim is followed immediately.

**Remark:**

If the direction fires the inversion in the points $M$ and $N$, then circle $p'$ is determined by the non-collinear points $O, M,$ and $N$ (Figure 3); and if the direction $p$ enters the circle of inversion in point $D$, then $p'$ is a circle with diameter $OD$ (Figure 4).

In order to demonstrate the following inversion properties, it is necessary to introduce the notion of potency in this respect in view of a crochet. First, we prove the following theorem.

**Theorem 4:**

Let $k(S, r)$ be a circle, $T$ be any plane, and $q$ be any direction that passes through $T$, and $A$ and $B$ be some points on the circle, then the product $p = |TA| \cdot |TB|$ does not depend on the choice of the direction $q$ through $T$, and we call it the potential $p$ point $T$ with respect to circle $k$.

**Proof 1:**

Let $T$ be a point on circle $k(S, r)$ (Figure 5), then $T = A$ or $T = B$. If $T = A$, then $|TA| = 0$; and if $T = B$, then $|TB| = 0$. From this, it follows that $|TA| \cdot |TB| = 0$.

**Proof 2:**

Let $T$ be a point within circle $k(S, r)$. Let $q_1$ and $q_2$ be the directions passing through point $T$, that is, $q_1 \cap k = \{A, B\}$ and $q_2 \cap k = \{C, D\}$ (Figure 6). Since the circumferential corners over the same arc are the same, it follows that:

$$\angle CAB = \angle CDB;$$
$$\angle DCA = \angle DBA.$$
It follows that triangles $TAC$ and $TDB$ are similar, and hence,

$$\frac{|TA|}{|TD|} = \frac{|TC|}{|TB|},$$

therefore, $|TA| \cdot |TB| = |TC| \cdot |TD|.$

**CONCLUSION**

After all, it remains to us only to identify a couple of obvious things. First of all, geometric inversion in relation to the circle, with its many and very affected properties, has a wide range of applications in non-standard geometric problems. When using invariants during the mapping itself, many difficult detectable dependencies between circles become apparent as additions between the real ones. In addition, we also presented the application of inversion in real life, which was found in an attempt to turn the circular motion into a straight line motion.

**BIBLIOGRAPHY**


**CITATION OF THE PAPER**

In this paper, after considering the properties of inversion, Feuerbach’s theorem and analytical proof of the theorem are given. Feuerbach’s theorem, one of the most beautiful theorems on the geometry of a triangle, states that the Euler circle of a triangle touches a triangle inscribed circle inside and all three triangles assigned a circle outside. The paper discusses the solution of the problem in an algebraic way and shows how this problem can be solved using inversion.

Therefore, inversion is used for various constructions related to circles so that by applying the inversion some of the circles are mapped in directions and thus the task is reduced to an analog simpler problem in which some circles are replaced by directions.